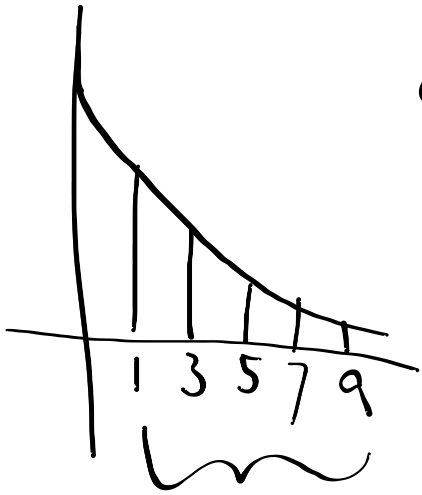
6.4/30 $f(x)=\frac{1}{x}$; $a=1$; $b=9$



$\frac{9-1=8}{4}=2$

a) left endpt: $A \approx 2\left[\frac{1}{1}+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}\right]$

$\{1,3,5,7\}$

b) midpt: $A \approx 2\left[\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}\right]$

$\{2,4,6,8\}$

c) right endpt: $A \approx 2\left[\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\frac{1}{9}\right]$

$\{3,5,7,9\}$

$$\frac{1}{1}+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}=\sum_{K=1}^{4}\frac{1}{2K-1}$$

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}=\sum_{K=1}^{4}\frac{1}{2K}$$

$$\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\frac{1}{9}=\sum_{K=1}^{4}\frac{1}{2K+1}=\sum_{K=2}^{5}\frac{1}{2K-1}=$$

6.5) Definite Integral

This idea of "Net Signed Area"

$\left(\lim_{n \to \infty} \sum_{k=1}^{n} f(x_k^*) \Delta x\right)$ is so important, we are going to extend this idea

A partition of an interval is a collectⁿ of sub-intervals

The sub-int do NOT have to have the same length.

The mesh size of a partition is the maximum (largest) width of a sub-int in that partition.

Our limit then becomes . . . . .

$$\lim_{\max \Delta x_k \to 0} \boxed{\sum_{k=1}^{n} f(x_k^*)\Delta x_k} = \text{Riemann sum}$$

called a Riemann sum

$$\int_a^b f(x)\,dx$$

(called the DEFINITE INTEGRAL)

$\int f(x)\,dx$

is the

- antiderivative
- aka the indefinite integral
- family of f^{n}'s whose deriv. $= f(x)$

$\int_a^b f(x)\,dx$

is the

- Net signed area
- aka the Definite Integral
- a <u>NUMBER</u>

If $\int_a^b f(x)\,dx$ exists...

i.e. if $\lim\limits_{\max \Delta x_k \to 0} \sum\limits_{k=1}^{n} f(x_k^*) \Delta x_k^*$

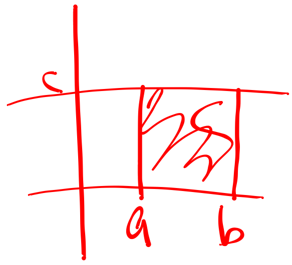
exists...

REGARDLESS of the partition...

the function is called INTEGRABLE

Some basic rules.

$$\int_a^b c\,dx = c(b-a)$$



$$\int_a^a f(x)\,dx = 0$$ [as long as a is in the domain of f(x)]

$$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$$

★ $$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

(regardless of order of a, b, c)

$$\int_a^b c f(x)\,dx = c\int_a^b f(x)\,dx$$

$$\int_a^b f(x) \pm g(x)\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

these are **NOT**

two-way facts

Differentiability

$\Downarrow$

Continuous

$\Downarrow$

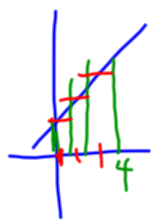
Integrable

$|x|$

Hw: 6.5/ 1, 5, 9, 11, 16

1) $f(x) = x+1$; $a=0$; $b=4$; $n=3$

$\Delta x_1 = 1$; $\Delta x_2 = 1$; $\Delta x_3 = 2$;



$x_1^* = \frac{1}{3}$, $x_2^* = \frac{3}{2}$; $x_3^* = 3$

a) $\sum_{k=1}^{n} f(x_k^*)\Delta x_k =$

b) $\max \Delta x_k =$

5) express as a definite integral

$$\lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n} (x_k^*)^2 \Delta x_k \; , a=-1 \; , b=2$$

9) express as a limit of Riemann sums (oops)

a) $\int_1^2 2x\,dx$

b) $\int_0^1 \frac{x}{x+1}\,dx$

6.4/29

$f(x) = 3x+1$; $a=2$; $b=6$

find $\sum_{k=1}^{4} f(x_k^*)\Delta x$

$\frac{6-2}{4} = 1$

a) left endpt

$x_k^* = \{2,3,4,5\}$

$f(2)\cdot 1 + f(3)\cdot 1 + f(4)\cdot 1 + f(5)\cdot 1$

$= 7\cdot 1 + 10\cdot 1 + 13\cdot 1 + 16\cdot 1$

$= \underline{46}$

b) midpt

$x_k^* = \{2.5, 3.5, 4.5, 5.5\}$

$= f(2.5)\cdot 1 + f(3.5)\cdot 1 + f(4.5)\cdot 1 + f(5.5)\cdot 1$

$= 8.5 + 11.5 + 14.5 + 17.5$

$= \underline{52}$

c) right endpt

$\{3,4,5,6\}$

$10+13+16+19 = 58$

6.5) "Net signed area" is an EXTREMELY IMPORTANT IDEA,

Recall, net signed area =

$$\lim_{n\to\infty} \sum_{k=1}^{n} f(x_k^*)\Delta x$$

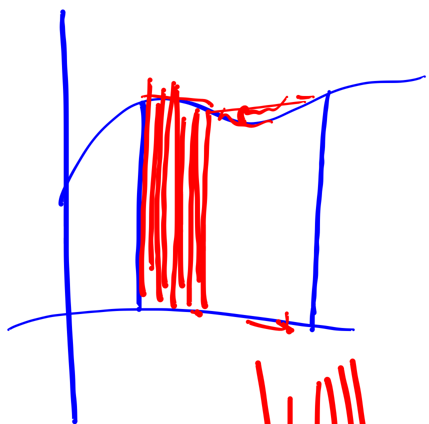
6.5 extends the idea of net signed area

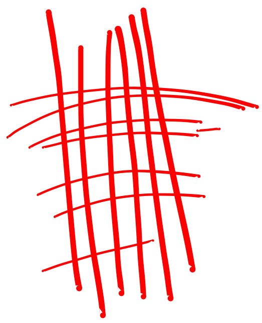
A *partition* of an interval is a collection of subintervals of the interval.

they don't have to be the same width

- every pt is in 1 subinterval
- sums of lengths of subintervals = interval

The *mesh size* of a partition is the width of the largest subinterval





So . . . .

$$\lim_{\max \Delta x_k \to 0} \boxed{\sum_{k=1}^{n} f(x_k^*)\,\Delta x_k}$$

called a Riemann sum

$$= \int_a^b f(x)\,dx$$

called the DEFINITE INTEGRAL

$\int f(x)\,dx$	$\int_a^b f(x)\,dx$
= antiderivative of $f(x)$	= definite integral
= indefinite integral	= net signed area (a limit of sums of areas)
= family of fns (with same shape)	= number

If $\int_a^b f(x)\,dx$ $\left(\substack{a \\ \text{limit}}\right)$ EXISTS...

NO MATTER WHAT the partition...

then the f^n $f(x)$ is called...

INTEGRABLE

these are NOT 2-way facts

DIFFERENTIABLE

$\Downarrow$

CONTINUOUS

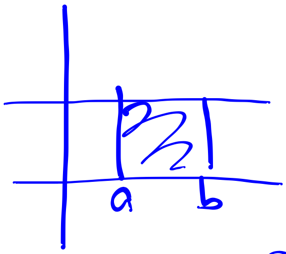
$\Downarrow$

INTEGRABLE

$|x|$

piecewise def^n

$$\int_a^b c\,dx = c(b-a)$$



$$\int_a^a f(x)\,dx = 0 \quad \left\{ \text{if } a \text{ is in domain of } f(x) \right\}$$

$$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$$

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

(no matter what order a, b, c are in)

$$\int_a^b c f(x)\,dx = c\int_a^b f(x)\,dx$$

$$\int_a^b f(x) \pm g(x)\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$